

Universality of Yukawa Couplings Confronts Recent Neutrino Data

G.C. Branco,¹ M.N. Rebelo,² and J.I. Silva-Marcos³

*Departamento de Física, Grupo Teórico de Física de Partículas,
Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisboa, Portugal.*

Abstract

We propose a flavour structure for the leptonic sector of the Standard Model, based on the idea of universality of Yukawa couplings, which accommodates all the experimental data on neutrino masses and mixing, at the same time predicting specific correlations between low energy measurable quantities, such as the ratio of neutrino squared mass differences, $|U_{13}|$, the leptonic Dirac phase, and the double-beta decay mass parameter. We also point out that it is possible, in this framework, to generate a sufficient amount of baryon asymmetry of the Universe through leptogenesis.

1 Introduction

The pattern of fermion masses and mixing remains a fascinating, unsolved puzzle. Within the Standard Model (SM) fermion masses and mixing are generated after gauge symmetry breaking; their flavour structure is arbitrary, since no symmetry of the SM constrains the flavour dependence of Yukawa couplings. The recent evidence for neutrino oscillations implies non-vanishing neutrino masses, which in turn requires an extension of the SM. Within the SM, neutrinos are strictly massless, since no Dirac masses are possible because of the absence of right-handed neutrinos, and no left-handed Majorana masses can be generated in higher orders owing to exact B-L conservation.

¹ On leave of absence at Theory Division, CERN, Geneva, Switzerland. E-mail address: gbranco@alfa.ist.utl.pt

² On leave of absence at Theory Division, CERN, Geneva, Switzerland. E-mail address: rebelo@alfa.ist.utl.pt

³ E-mail address: Joaquim.Silva-Marcos@cern.ch

The simplest extension of the SM, which leads to non-vanishing neutrino masses, consists of adding to the spectrum of the SM three right-handed neutrinos (one per generation). If no other symmetries are introduced, one is then naturally led to non-vanishing, but naturally small, neutrino masses through the see-saw mechanism.

It was suggested some time ago that the diversity of quark masses and mixing may be closely related to the fact that Yukawa couplings are the only couplings of the SM that can be complex. In particular, it was shown that the observed pattern of quark masses and mixing can be accounted for within the framework of the Universality of Strength of Yukawa couplings (USY), where Yukawa couplings all have the same modulus, the flavour dependence being all contained in the phases [1]. This leads to mass matrices of the form: $M_q = c_q[\exp(i\theta_{jk}^q)]$. These mass matrices have very interesting properties [2]–[5], and it was shown [6] that such a pattern can be generated in the context of extra dimensions with fermions localized in a thick brane. The USY hypothesis was extended to the leptonic sector in previous works, and in particular it was shown in [7] that it is possible within USY to obtain the large mixing angle solar solution (LMA) together with large atmospheric mixing.

Constraints on the structure of the neutrino mass matrix and CP violation and their implications can be analysed [8], based solely on the current data available from experiments with solar [9], [10] and atmospheric [11] neutrinos, reactor experiments [12], [13], neutrinoless double beta decay searches [14], astrophysics and cosmology [15]. If CP were conserved in the leptonic sector, all six of the independent entries of the effective low energy neutrino mass m_{eff} , resulting from the see-saw mechanism, could be determined from experiment, through the measurement of three neutrino masses and three mixing angles. If one assumes the general situation, one is led to leptonic CP violation at low energies, with a 3×3 complex symmetric effective neutrino mass matrix, corresponding to six independent moduli and three phases. It has been emphasized [16] that this leads to a situation where *no currently conceivable set of feasible experiments can fully determine the neutrino mass matrix*. This is because realistic experiments can only measure seven independent quantities, to wit Δm_{12}^2 , Δm_{23}^2 , θ_{12} , θ_{13} , θ_{23} , δ , $|\langle m \rangle|$ (i.e. two squared neutrino mass differences, three mixing angles, the Dirac-type CP violating phase and the effective mass element relevant to neutrinoless double beta decay, which is sensitive to the presence of Majorana phases). This observation led to various suggestions to limit the number of parameters in the neutrino mass matrix, such as the possibility that the neutrino mass matrix has texture zeros [16], [17]. An alternative proposal, to remove the ambiguities in the reconstruction of the neutrino mass matrix, consists of imposing the weak basis (WB)-independent condition $\det(m_{\text{eff}}) = 0$, without imposing texture zeros [18]. In this paper we apply the USY hypothesis together with the condition that the determinant of m_{eff} is zero, to construct a highly predictive scheme

where not only the low energy neutrino mass matrix can be reconstructed, but we also obtain specific predictions both for low energy phenomena and for leptogenesis [19].

2 Framework

2.1 Choice of *USY* ansatz

We add to the spectrum of the SM three right-handed neutrino fields ν_R^0 (one per generation), which leads, after spontaneous symmetry breaking, to the following mass terms:

$$\begin{aligned}\mathcal{L}_m &= - \left[\overline{\nu_L^0} m \nu_R^0 + \frac{1}{2} \nu_R^{0T} C M \nu_R^0 + \overline{l_L^0} m_l l_R^0 \right] + \text{h.c.} = \\ &= - \left[\frac{1}{2} n_L^T C \mathcal{M}^* n_L + \overline{l_L^0} m_l l_R^0 \right] + \text{h.c.} ,\end{aligned}\tag{1}$$

where m , M and m_l denote the neutrino Dirac mass matrix, the right-handed neutrino Majorana mass matrix and the charged lepton mass matrix, respectively, $n_L = (\nu_L^0, (\nu_R^0)^c)$ (written here as a line instead of a column in order to save space) and \mathcal{M} is given by

$$\mathcal{M} = \begin{pmatrix} 0 & m \\ m^T & M \end{pmatrix} .\tag{2}$$

The zero entry in \mathcal{M} reflects the absence of a term of the form $\frac{1}{2} \nu_L^{0T} C m_L \nu_L^0$, which requires, in order to be generated at tree level, an extension of the Higgs sector of the SM. The matrix M corresponds to an $SU(2) \times U(1) \times SU(3)_c$ invariant mass term; therefore its entries can naturally be of a scale much higher than the electroweak scale. Under this assumption, the light neutrino masses are obtained, to an excellent approximation, from the diagonalization of the effective neutrino mass matrix defined as $m_{\text{eff}} \equiv -m M^{-1} m^T$:

$$- V^\dagger m M^{-1} m^T V^* = d_\nu .\tag{3}$$

In this work we suggest the following specific USY ansatz for the three different mass matrices appearing in \mathcal{L}_m :

$$m_l = c_l K \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{ia_l} & 1 \\ 1 & 1 & e^{ib_l} \end{bmatrix}, \quad m = c_D \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & e^{ia} \\ 1 & 1 & e^{ib} \end{bmatrix}, \quad (4)$$

$$M = c_R \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{iA} & 1 \\ 1 & 1 & e^{iB} \end{bmatrix},$$

where $K \equiv \text{diag}(1, 1, e^{i\theta})$ and c_l , c_D , and c_R are real constants, i.e. we assume that it is possible to cast all of the leptonic mass matrices simultaneously in a USY form. In order to implement the condition $\det(m_{\text{eff}}) = 0$, we have chosen the Dirac mass matrix m to have the USY form, but with vanishing determinant. Within the USY framework, all inequivalent forms leading to a vanishing determinant have been classified, and it was shown that there are only two classes of solutions [4]. The Majorana mass matrix is also of the USY form and it has the nice feature that its inverse has a simple form. From Eq. (4) one obtains:

$$M^{-1} = \frac{1}{N} \begin{bmatrix} e^{i(A+B)} - 1 & 1 - e^{iB} & 1 - e^{iA} \\ 1 - e^{iB} & e^{iB} - 1 & 0 \\ 1 - e^{iA} & 0 & e^{iA} - 1 \end{bmatrix}; \quad N = c_R(e^{iA} - 1)(e^{iB} - 1). \quad (5)$$

It is useful to write m as:

$$m = c_D(\Delta + P), \quad (6)$$

with

$$P = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & e^{ia} - 1 \\ 0 & 0 & e^{ib} - 1 \end{bmatrix} \quad \text{and} \quad \Delta = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}. \quad (7)$$

From Eqs. (5), (6) and (7), and using the fact that $\Delta M^{-1}P + PM^{-1}\Delta = 0$, we obtain:

$$-m_{\text{eff}} \equiv m \frac{1}{M} m^T = c_{\text{eff}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 + kx^2 & kxy \\ 1 & 1 + kxy & 1 + ky^2 \end{bmatrix}, \quad (8)$$

with $k = \frac{1}{e^{iB}-1}$, $x = e^{ia} - 1$, $y = e^{ib} - 1$ and $c_{\text{eff}} = c_D^2/c_R$. Notice that the parameter A does not appear at low energies. In the following we show that $a, b \ll 1$, so that m_{eff} is a small perturbation around Δ .

The leptonic charged current interactions can then be written as:

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} \overline{l_L} \gamma_\mu U \nu_L W^\mu + \text{h.c.}, \quad (9)$$

where the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix [20] is given by $U = V_l^\dagger V$, with V_l denoting the unitary matrix entering in the diagonalization of the charged lepton mass matrix:

$$V_l^\dagger m_d m_d^\dagger V_l = d_l^2. \quad (10)$$

We use the standard parametrization [21] for U , factoring out the Majorana type phase

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot P, \quad (11)$$

where $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$, with all θ_{ij} in the first quadrant and $P = \text{diag}(1, 1, e^{i\alpha})$; δ is a Dirac-type phase (analogous to the one of the quark sector) and α is the physical phase associated with the Majorana character of neutrinos. In general, for three generations, there are two Majorana phases, but the fact that one of the light neutrino masses vanishes eliminates one of these phases.

2.2 Counting of parameters

In order to evaluate the predictive power of our ansatz, let us compare its number of parameters with the total number of parameters present in general

with one zero neutrino mass. We will do this comparison first taking into account the full parameter space, and later considering only the parameters that are relevant in the low energy limit.

2.2.1 The full parameter space

From Eq. (4) one sees that our ansatz is characterized by ten parameters namely (c_R, A, B) , (c_D, a, b) , (c_l, θ, a_l, b_l) . Let us now count the number of independent parameters characterizing the lepton masses and mixing in the general case, but with the implicit assumption that one light neutrino mass vanishes. The total number of parameters can be obtained by going to the WB where both m_l and M are real, diagonal. Then apart from the six real parameters characterizing the eigenvalues of m_l and M , one has to take into account the number of parameters needed for specifying, in the above WB, the Dirac neutrino mass matrix m , taking into account the vanishing of its determinant. In this case, it can be shown that any matrix m can be written as:

$$m = \hat{U}_L d P_1 \hat{U}_R P_2, \quad (12)$$

where $\hat{U}_L(\delta_L)$ $\hat{U}_R(\delta_R)$ are unitary matrices containing only one phase each (à la Kobayashi–Maskawa) and

$$d = \text{diag}(0, m_2, m_3); \quad P_1 = \text{diag}(1, e^{i\phi}, 1); \quad P_2 = \text{diag}(e^{i\sigma_1}, e^{i\sigma_2}, 1). \quad (13)$$

We have already omitted the three phases that can always be rephased away irrespective of the value of $\det(m)$. From Eqs. (12) and (13) it follows that m is characterized by thirteen parameters, consisting of eight real numbers and five phases. Together with the six eigenvalues of m_l and M , one has altogether nineteen parameters needed to specify the three leptonic mass matrices m , m_l and M . This equals the number of physical quantities (eight non-zero masses, six mixing angles, five CP violating phases). The condition $\det(m_{\text{eff}}) = 0$ eliminates one mass and one phase, so that in general with see-saw one would have twenty one parameters [22]. This is to be compared with the ten parameters characterizing our ansatz, thus showing its high predictive power. In general, the phases appearing in m generate CP violation both at low and at high energies, where they are essential to produce viable leptogenesis. Furthermore, it has been pointed out that these CP violating phases may manifest themselves at low energies even without CP violation at high energies [23]; conversely it is possible to have CP violation at high energies with no CP violation at low energies [24] either of Dirac or of Majorana type. Although the above counting is useful in gauging the predictive power of our ansatz, it is clear that at present we can only expect to be able to measure low energy

parameters. Therefore, it is interesting to make an analogous counting, taking only those into account, as well as to discuss how the low energy parameters are fixed.

2.2.2 The low energy limit and fixing the parameters

At low energies our ansatz is characterized by eight parameters, namely, θ , c_l , a_l , b_l , c_{eff} , a , b , B , whilst ten parameters are needed to parametrize low energy data, namely m_e , m_μ , m_τ , and m_2 , m_3 (the two non-vanishing light neutrino masses), and also θ_{12} , θ_{13} , θ_{23} , δ , and α , which characterize the PMNS matrix.

The three independent real parameters c_l , a_l and b_l in m_l are fixed once we impose the correct values for the charged lepton masses. Taking into account the charged lepton mass hierarchy, we find the following approximate relations [25]:

$$|a_l| \simeq 6 \frac{m_e}{m_\tau}, \quad |b_l| \simeq \frac{9}{2} \frac{m_\mu}{m_\tau}, \quad |c_l| \simeq \frac{m_\tau}{3}. \quad (14)$$

With all four parameters of m_l fixed, the diagonalizing matrix V_l is then determined [25]:

$$V_l = K \cdot F \cdot W \quad ; \quad F = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}, \quad (15)$$

$$K \equiv \text{diag}(1, 1, e^{i\theta})$$

where for W one has, to leading order:

$$W \simeq \begin{pmatrix} 1 & \frac{m_e}{\sqrt{3}m_\mu} & -i\sqrt{\frac{2}{3}} \frac{m_e}{m_\tau} \\ -\frac{m_e}{\sqrt{3}m_\mu} & 1 & i\frac{m_\mu}{\sqrt{2}m_\tau} \\ -i\sqrt{\frac{3}{2}} \frac{m_e}{m_\tau} & i\frac{m_\mu}{\sqrt{2}m_\tau} & 1 \end{pmatrix}. \quad (16)$$

The four parameters a , b , B and $c_{\text{eff}} \equiv c_D^2/c_R$ appearing in the effective neutrino mass matrix are directly related to the light neutrino squared mass differences. We obtain for the invariants (taking into account that $m_1 = 0$) $\chi \equiv \chi(h) = m_2^2 m_3^2$ and the trace $t \equiv \text{Tr}(h) = m_2^2 + m_3^2$ of the hermitian matrix

$h \equiv m_{\text{eff}} m_{\text{eff}}^\dagger$, expressed as functions of a , b , B and c_{eff} :

$$\begin{aligned} \frac{\chi}{c_{\text{eff}}^4} &= 4 \left[\sin^2\left(\frac{a}{2}\right) + \sin^2\left(\frac{b}{2}\right) + \sin^2\left(\frac{a-b}{2}\right) \right]^2 / \sin^2\left(\frac{B}{2}\right) \\ \frac{t}{c_{\text{eff}}^2} &= 9 + [3 + \cos(a-b) + \cos(a+b-B) + \cos(a-b)\cos(a+b-B) \\ &\quad + 2\cos(B) - 2\cos(a) - 2\cos(b) - 2\cos(a-B) - 2\cos(b-B)] / \sin^2\left(\frac{B}{2}\right) . \end{aligned} \quad (17)$$

Thus, assuming small values for a , b and B , one finds the approximate relations:

$$\chi \simeq c_{\text{eff}}^4 \frac{4}{B^2} (a^2 + b^2 - ab)^2, \quad (18)$$

$$t \simeq 9 c_{\text{eff}}^2, \quad |c_{\text{eff}}| \simeq m_3/3 \quad (19)$$

and for the neutrino mass ratio $r \equiv \frac{m_2}{m_3}$:

$$\frac{\sqrt{\chi}}{t} = \frac{r}{1+r} \simeq r \simeq \sqrt{\frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2}} \simeq \frac{2}{9B} (a^2 + b^2 - ab) . \quad (20)$$

Note that for a fixed mass ratio r and for a fixed B , this formula constrains the values of a and b to an (almost) ellipse.

It is also possible to obtain simple approximate expressions for θ_{atm} and θ_{sol} . Assuming a and b “small”, $\sin^2(2\theta_{\text{atm}})$ is, to leading order, only a function of θ , and one obtains:

$$\sin^2(2\theta_{\text{atm}}) \simeq 4 \cdot p (1-p) \quad ; \quad p = \frac{8}{9} \sin^2\left(\frac{\theta}{2}\right) , \quad (21)$$

whilst $\sin^2(2\theta_{\text{sol}})$ is approximately given by:

$$\sin^2(2\theta_{\text{sol}}) \simeq 4 \cdot q (1-q) \quad ; \quad q = \frac{3a^2}{4(a^2 + b^2 - ab)} . \quad (22)$$

It is clear from Eqs. (20) and (22) that, in the range of validity of these expressions, physics at low energies only puts constraints on the ratios a/\sqrt{B} , b/\sqrt{B} , and not on a , b and B separately. Another interesting point is the fact that for $\theta_{\text{sol}} = 30^\circ$, Eq. (22) has two very simple solutions of the form $b = 2a$ and $b = -a$, corresponding to straight lines in the (a, b) plane. In the next section the currently allowed ranges for Δm_{21}^2 , Δm_{32}^2 , $\tan^2(\theta_{12})$ ($\theta_{12} \equiv \theta_{\text{sol}}$)

and $\tan^2(\theta_{23})$ ($\theta_{23} \equiv \theta_{\text{atm}}$) are given; it can be seen that this value for θ_{sol} is close to the central allowed experimental value. The numerical search for the allowed parameter space was performed with exact expressions and led to regions in the (a, b) plane in the neighbourhood of the above straight lines.

For hierarchical heavy neutrino masses, it is clear that A and B are small and verify relations similar to those for a_l and b_l :

$$|A| \simeq 6 \frac{M_1}{M_3}, \quad |B| \simeq \frac{9}{2} \frac{M_2}{M_3}, \quad A, B \ll 1. \quad (23)$$

The fact that one of the light neutrino masses is 0 constrains c_{eff} to be of the order of $\sqrt{\Delta m_{\text{atm}}^2} \sim 10^{-2}$ eV. Taking c_D to be of the electroweak scale ($\sim 10^{11}$ eV) one obtains c_R of order $\sim 10^{16}$ GeV, implying M_3 of the same order. Heavy neutrino masses are not further constrained by experiment. The value of the parameter A does not affect low energy physics, since it does not appear in the exact expression for m_{eff} , given in Eq. (8). In this framework, neither $|U_{13}|$ nor δ and α are free parameters. It is possible to have $|U_{13}|$ close to its experimental limit and the CP violating parameter I defined by $|\text{Im}[U_{ij}U_{kl}U_{kj}^*U_{il}^*]|$, which is sensitive to the value of δ , can be of order 10^{-2} , within the reach of future neutrino experiments, meaning that in our framework the phase δ can be large.

In the next section we present the implications of our model for low energy physics.

3 Examples and predictions

It was shown in section 2 that our USY model has fewer free parameters than the total number of measurable quantities at low energies. Therefore, the model predicts specific correlations among these quantities. In this section, we first describe how the analysis was performed and present a specific interesting example, which is taken up again in the discussion of leptogenesis. Next, we present, in the form of various plots, correlations between physical quantities, which are implied by our framework, and comment on the main features of our model.

3.1 Strategy

In our analysis, illustrated in part by the graphs that follow, we have fixed the parameters c_l , a_l and b_l in such a way that the PDG values [21] given

for the charged lepton masses are obtained. The masses of $m_e = 0.511$ MeV, $m_\mu = 106$ MeV and $m_\tau = 1777$ MeV correspond to $a_l = 1.725089 \times 10^{-3}$, $b_l = 0.26785$ and $c_l = 593.3863$ MeV. With these values the leptonic mixing matrix is obtained, still with θ as a free parameter (since factorizable phases do not affect the eigenvalues of the mass matrix). The resulting matrix V_l is given by:

$$V_l = K \cdot \begin{bmatrix} 0.70824 & -0.40554 & 0.57787 \\ -0.70597 - 0.00031i & -0.40949 - 0.00006i & 0.57786 + 0.00033i \\ -0.00227 - 0.00030i & 0.81400 + 0.07250i & 0.57404 + 0.05125i \end{bmatrix}. \quad (24)$$

As expected, this matrix is very close to the matrix F in Eq. (15).⁴ At low energies, the remaining relevant parameters are c_{eff} , a , b , B and θ . In our search for allowed regions of parameters we have scanned θ from 0 to 2π , a and b in the region $[-0.4, 0.4]$, and B in the interval $[-0.021, 0.021]$. Notice that B is chosen to be much smaller than 1, so that heavy neutrinos have hierarchical masses (these also depend on A). It follows from Eq. (20) that, in this case, the values of a and b compatible with the experimental constraints on neutrino masses are well inside the interval given above for these parameters. For each fixed set of parameters, the ratio $\Delta m_{21}^2/\Delta m_{32}^2$ as well as $\tan^2(\theta_{12})$ and $\tan^2(\theta_{23})$ were determined and compared with the currently allowed experimental ranges, consistent with the first results from K2K and KamLAND, as given in [26] (notice that θ_{12} is the solar angle and θ_{23} the atmospheric angle):

$$\begin{aligned} (1.5) \quad & 2.2 < \Delta m_{32}^2/10^{-3} \text{ eV}^2 < 3.0 \quad (3.9) \\ (0.45) \quad & 0.75 < \tan^2(\theta_{23}) < 1.3 \quad (2.3) \\ (5.4) \quad & 6.7 < \Delta m_{21}^2/10^{-5} \text{ eV}^2 < 7.7 \quad (10.0) \quad \text{and} \quad (25) \\ & (14.0) < \Delta m_{21}^2/10^{-5} \text{ eV}^2 < (19.0) \\ (0.29) \quad & 0.39 < \tan^2(\theta_{12}) < 0.51 \quad (0.82) . \end{aligned}$$

These are 1σ (3σ) CL intervals; the second range of Δm_{21}^2 corresponds to solutions in the upper LMA island (at present the results of the solar and KamLAND analyses still allow for an ambiguity in the determination of Δm_{21}^2 at CL $> 2.5\sigma$). Furthermore in our programme we have imposed a cut in $|U_{13}|^2 \equiv \sin^2(\theta_{13}) < 0.02$. We have used 1σ CL intervals for the solar and atmospheric angles; for the mass ratio we worked with a 3σ CL, since it is a feature of our model that solutions with $|U_{13}|$ close to the experimental

⁴ Five decimal digits are quite sufficient for the computation of the matrix U , which is not yet known experimentally with such high precision. The check of the eigenvalue equation for the charged lepton masses requires that we know V_l with much higher precision since the squared masses of the charged leptons differ by many orders of magnitude.

bound and leading to a large value of $|I|$ correspond to values of Δm_{21}^2 below $6.7 \times 10^{-5} \text{ eV}^2$. Our ansatz predicts that sizable CP violation in neutrino oscillations is not compatible with the upper LMA.

The numerical analysis was performed with exact formulae. Since we constrain the ratio of masses rather than the masses themselves, we do not need to fix c_{eff} . Let us, for the sake of illustration, present a possible solution in the region $b \simeq 2a$, with both a and b positive:

$$a = 0.058998; \quad b = 0.102998; \quad \theta = 1.9 \text{ rad} . \quad (26)$$

In this example we obtain, fixing $c_{\text{eff}} = 0.0163 \text{ eV}$:

$$\begin{aligned} \Delta m_{21}^2 &= 6.6 \times 10^{-5} \text{ eV}^2, \quad \Delta m_{32}^2 = 2.7 \times 10^{-3} \text{ eV}^2, \\ \tan^2(\theta_{12}) &= 0.46, \quad \tan^2(\theta_{23}) = 0.99 \\ |U_{13}| &= 0.13 \text{ and } |I| = 0.02 . \end{aligned} \quad (27)$$

This example has large $|U_{13}|$, still compatible with the experimental bound [26] of $\sin^2(\theta_{13}) < 0.02$ (0.052) at 1σ (3σ) and large $|I|$, within the reach of the neutrino experiments being planned. This is a generic characteristic of the examples in the region around $b \simeq 2a$ and θ close to 1.9 rad. This example is taken again in the discussion of leptogenesis.

3.2 Correlations and implications for low energy CP violation

The simplest way of describing the predictions of our USY model is by presenting the correlations implied by it, for various physical quantities. Our results are depicted in Figs. 1–5. The points correspond to solutions satisfying all the experimental constraints. We have chosen to depict correlations between physical observables rather than between the initial parameters of the Lagrangian. There are two different disconnected regions for $|U_{13}|^2$, once all other experimental constraints are imposed, one of them characterized by very small values of $|U_{13}|^2$, which are more than one order of magnitude below its upper bound, obtained around $b \simeq -a$, and the other around $b \simeq 2a$, with values of the order of magnitude of the upper bound. Only in the region of larger $|U_{13}|^2$ can one expect to observe CP violation at the neutrino factories being planned at present.

In Fig. 1 we present the correlation between $|U_{13}|^2$ and $\tan^2(2\theta_{\text{sol}})$. It is clear that there are two regions corresponding respectively to small and relatively large values of $|U_{13}|^2$, as mentioned above. However, in each one of these regions, $\tan^2(2\theta_{\text{sol}})$ may have any value within the experimental bound. In

Fig. 2, we present the correlation between $r \equiv (\Delta m_{21}^2/\Delta m_{32}^2)^{1/2}$ and $|U_{13}|^2$. Again, we find two regions corresponding to smaller and larger values of $|U_{13}|^2$. Furthermore, Fig. 2 clearly shows that in the present model, relatively large values of $|U_{13}|^2$ are only possible for small values of the mass ratio r . In Fig. 3 we plot the correlation between $|U_{13}|^2$ and the CP-odd rephasing invariant I . In terms of mixing angles, I is given by:

$$I = \frac{1}{8} \sin(2\theta_{12}) \sin(2\theta_{23}) \sin(2\theta_{13}) \cos(\theta_{13}) \sin \delta ; \quad (28)$$

from this expression it follows that a “large” value of I (e.g. $I \sim 10^{-2}$) requires $|U_{13}|$ close to its upper bound and an unsuppressed Dirac phase δ . It can be clearly seen that there are many solutions corresponding to a large Dirac phase. For I of the order of 10^{-2} , leptonic CP violation may be detected at neutrino factories through the study of neutrino oscillations. In Fig. 4 we present the allowed regions of I versus r . It is clear that relatively large values of I are only possible for small values of r . This result was to be expected, since large values $|U_{13}|$ are only obtained for regions of small r .

3.3 Implications for double beta decay

In Fig. 5 the correlation between $(\delta - \alpha)$ (the phase appearing in U_{13} as defined in Eq. (11)), and $|U_{13}|^2$ is given. Neutrinoless double beta decay measures $|\langle m \rangle|$ defined by:

$$|\langle m \rangle| \equiv |m_1 U_{11}^2 + m_2 U_{12}^2 + m_3 U_{13}^2| = |m_2 s_{12}^2 c_{13}^2 + m_3 s_{13}^2 e^{2i(\delta - \alpha)}| , \quad (29)$$

where we have used in the second equality the fact that in our ansatz m_1 vanishes. It is clear that this observable is sensitive to the combination of phases $(\delta - \alpha)$. In models with $m_1 = 0$, the term $m_2 s_{12}^2 c_{13}^2$ is of order 10^{-3} eV; on the other hand the product $m_3 s_{13}^2$ can only reach this order of magnitude for s_{13}^2 around its maximal experimental bound of order 10^{-2} . In this case, cancellations might occur for $(\delta - \alpha)$ close to $\pi/2$ rad. Yet Fig. 5 shows this phase to be somewhat below $\pi/4$ rad in the region of larger $|U_{13}|^2$, thus rendering cancellations impossible. As a result our framework predicts $|\langle m \rangle|$ of order 10^{-3} eV, which is somewhat below the range favoured by the Moscow–Heidelberg experiment [14], but within the reach of the next generation of experiments.

4 Leptogenesis in this framework

It is well known that, in the case of hierarchical heavy neutrinos, the baryon asymmetry generated through thermal leptogenesis only depends on four parameters [27]: the mass M_1 of the lightest heavy neutrino, together with the corresponding CP asymmetry ϵ_{N_1} in their decays, as well as the effective neutrino mass \widetilde{m}_1 defined as

$$\widetilde{m}_1 = (m^\dagger m)_{11}/M_1 \quad (30)$$

in the weak basis where M is diagonal, real and positive and, finally, the sum of all light neutrino masses squared, $\overline{m}^2 = m_1^2 + m_2^2 + m_3^2$, which controls an important class of washout processes.

The CP asymmetry ϵ_{N_1} generated by the lightest heavy neutrino is explicitly given by [28]:

$$\epsilon_{N_1} = \frac{1}{8\pi v^2} \frac{1}{(m^\dagger m)_{11}} \sum_{i=2,3} \text{Im} \left[(m^\dagger m)_{1i}^2 \right] \left[f\left(\frac{M_i^2}{M_1^2}\right) + g\left(\frac{M_i^2}{M_1^2}\right) \right], \quad (31)$$

where $v = \langle \phi^0 \rangle / \sqrt{2} \simeq 174 \text{ GeV}$ and $f(x)$, $g(x)$ denote the one-loop vertex and self-energy corrections:

$$f(x) = \sqrt{x} \left[1 + (1+x) \ln \left(\frac{x}{1+x} \right) \right], \quad g(x) = \frac{\sqrt{x}}{1-x}. \quad (32)$$

In the limit $M_1 \ll M_2, M_3$, the CP asymmetry (31) can be written as:

$$\epsilon_{N_1} \simeq -\frac{3}{16\pi v^2} \left(I_{12} \frac{M_1}{M_2} + I_{13} \frac{M_1}{M_3} \right), \quad (33)$$

where

$$I_{1i} \equiv \frac{\text{Im} \left[(m^\dagger m)_{1i}^2 \right]}{(m^\dagger m)_{11}}. \quad (34)$$

The lepton asymmetry Y_L is connected to the CP asymmetry through the relation [29]

$$Y_L = \frac{n_L - n_{\bar{L}}}{s} = d \frac{\epsilon_{N_1}}{g_*}, \quad (35)$$

where g_* is the effective number of relativistic degrees of freedom contributing to the entropy and d is the so-called dilution factor, which accounts for the washout processes (inverse decay and lepton number violating scattering). In the SM case, $g_* = 106.75$.

The produced lepton asymmetry Y_L is converted into a net baryon asymmetry Y_B through the $(B + L)$ -violating sphaleron processes [30]. One finds the relation [31], [32]:

$$Y_B = \xi Y_{B-L} = \frac{\xi}{\xi - 1} Y_L \quad , \quad \xi = \frac{8 N_f + 4 N_H}{22 N_f + 13 N_H} \quad , \quad (36)$$

where N_f and N_H are the number of fermion families and complex Higgs doublets, respectively. Taking into account that $N_f = 3$ and $N_H = 1$ for the SM, we get $\xi \simeq 1/3$ and

$$Y_B \simeq -\frac{1}{2} Y_L \quad . \quad (37)$$

Successful leptogenesis would require ϵ_{N_1} of order 10^{-8} if washout processes could be neglected (i.e. $d = 1$), in order to reproduce the observed ratio of baryons to photons, which is given by [15]:

$$\frac{n_B}{n_\gamma} = (6.1^{+0.3}_{-0.2}) \times 10^{-10} \quad . \quad (38)$$

Leptogenesis is a non-equilibrium process that takes place at temperatures $T \sim M_1$. This imposes an upper bound on the effective neutrino mass \widetilde{m}_1 given by the “equilibrium neutrino mass” [29], [33], [34]:

$$m_* = \frac{16\pi^{5/2}}{3\sqrt{5}} g_*^{1/2} \frac{v^2}{M_{Pl}} \simeq 10^{-3} \text{ eV} \quad , \quad (39)$$

where M_{Pl} is the Planck mass ($M_{Pl} = 1.2 \times 10^{19}$ GeV). The sum of all neutrino masses squared \overline{m}^2 is constrained, in the case of normal hierarchy, to be below 0.20 eV [35]. This bound is automatically verified in the case $m_1 = 0$.

In the WB, where M is diagonal, the product $m^\dagger m$ is transformed, in our framework, into:

$$m^\dagger m \rightarrow (F \cdot W_R \cdot K_R)^T m^\dagger m (F \cdot W_R \cdot K_R)^* \quad , \quad (40)$$

corresponding to the transformation:

$$\nu_R^0 \rightarrow (F \cdot W_R \cdot K_R)^* \nu_R^0 \quad , \quad (41)$$

where F denotes the matrix given in Eq. (15), K_R is of the form $K_R = \text{diag}(-e^{i\pi/4}, e^{i\pi/4}, 1)$ and W_R is similar to W in Eq. (16), with the masses of charged leptons replaced by the masses of heavy neutrinos. Expanding W_R up to the next order in the ratios of masses leads to:

$$W_R \simeq \begin{pmatrix} 1 + \frac{9i}{8} \frac{M_1}{M_3} & -\frac{M_1}{\sqrt{3}M_2} \left(1 + \frac{i}{8} \frac{M_2}{M_3}\right) & -i\sqrt{\frac{2}{3}} \frac{M_1}{M_3} \left(1 + \frac{i}{4} \frac{M_2}{M_3}\right) \\ \frac{M_1}{\sqrt{3}M_2} \left(1 + \frac{3i}{4} \frac{M_2}{M_3}\right) & 1 + \frac{7i}{8} \frac{M_2}{M_3} & -\frac{i}{\sqrt{2}} \frac{M_2}{M_3} \left(1 + i \frac{M_2}{M_3}\right) \\ -i\sqrt{\frac{3}{2}} \frac{M_1}{M_3} & -\frac{i}{\sqrt{2}} \frac{M_2}{M_3} \left(1 + \frac{i}{8} \frac{M_2}{M_3}\right) & 1 + \frac{i}{4} \frac{M_2}{M_3} \end{pmatrix}. \quad (42)$$

From Eq. (40) it is possible to show that the leading contributions to the different matrix elements of $m^\dagger m$, which appear in the formulae relevant to leptogenesis are given by:

$$(m^\dagger m)_{11} = -\frac{2}{9} c_D^2 \left(\frac{M_1}{M_2}\right)^2 \left[a^2 + b^2 - 9(a+b) \left(\frac{M_2}{M_3}\right)\right] \quad (43)$$

$$\text{Re}[(m^\dagger m)_{12}] = \frac{2}{3\sqrt{3}} c_D^2 \left(\frac{M_1}{M_2}\right) \left[a^2 + b^2 - 6(a+b) \left(\frac{M_2}{M_3}\right)\right] \quad (44)$$

$$\text{Im}[(m^\dagger m)_{12}] = \frac{-5}{12\sqrt{3}} c_D^2 \left(\frac{M_1}{M_3}\right) \left[(a^2 + b^2) - \frac{24}{5}(a+b) \left(\frac{M_2}{M_3}\right)\right] \quad (45)$$

$$(m^\dagger m)_{13} = e^{-i\pi/4} \sqrt{\frac{2}{3}} c_D^2 \left(\frac{M_1}{M_2}\right) \left[(a+b) - \frac{27}{2} \left(\frac{M_2}{M_3}\right) - \frac{i}{6}(a^2 + b^2)\right]. \quad (46)$$

As an example, for $M_1 \sim 10^{10}$ GeV and $M_2 \sim 10^{13}$ GeV, which implies fixing the parameters A and B , together with M_3 , which is fixed to be of order 10^{16} GeV as explained in Section 2, we obtain:

$$\widetilde{m}_1 \sim 10^{-5} \text{ eV}, \quad \epsilon_{N_1} \sim 10^{-7}, \quad (47)$$

where we have chosen $a = 0.052998$, $b = 0.102998$ and $B = 0.01$, as in the explicit example of the previous section, together with $A = 6 \times 10^{-6}$ and $c_R = 3 \times 10^{24}$ eV. This leads to the observed baryon asymmetry of the Universe, assuming a washout factor of order 10^{-1} .

Expressions (43) to (46) can be rewritten in terms of initial parameters A , B and a, b with the help of Eq. (23). Since A is not constrained by the low energy physics, there is still a free variable parametrizing leptogenesis. Varying A corresponds to different choices of M_1 , which, as was pointed out in the case of hierarchy of heavy neutrino masses, is the most important of the three masses. On the other hand, low energy physics fixes the ratios $\frac{a}{\sqrt{B}}$, $\frac{b}{\sqrt{B}}$ for variable B .

5 Conclusions

We have investigated the pattern of lepton masses and mixing within the framework of USY, where all Yukawa couplings have equal modulus, the flavour structure being all contained in the phases of those couplings.

In our scheme, three of the four parameters of the charged lepton mass matrix m_l are related to the three charged lepton masses, the fourth one (the phase θ) giving the dominant contribution to $\sin^2(2\theta_{\text{atm}})$; the three parameters in the Dirac neutrino mass matrix m together with c_R are constrained by the two neutrino squared mass differences (for each choice of B) and by $\sin^2(2\theta_{\text{sol}})$. Furthermore, the assumption that c_D is of the order of the weak scale fixes the scale of the heavy neutrino masses and, with our choice of hierarchical heavy neutrino masses, it determines, to an excellent approximation, the mass of the heaviest neutrino. The remaining parameters A and B are dependent on the hierarchy of the heavy neutrino masses, for which there is no experimental information available. It should be noticed that A has no influence on low energy physics. Strong hierarchy of heavy neutrino masses requires A and B to be much smaller than 1. Hierarchical heavy neutrinos are generally favoured in scenarios of baryogenesis through leptogenesis (together with one light neutrino with zero mass), since in this case the washout processes tend to be less important.

In conclusion, we have suggested an ansatz for the leptonic flavour structure, where all the experimental data on lepton masses and mixing are reproduced and specific correlations between measurable quantities are obtained, such as U_{13} , the ratio of neutrino squared mass differences, the strength of leptonic Dirac-type CP violation and the mass parameter $|\langle m \rangle|$, measurable in neutrinoless double beta decay. In particular, in the scheme we propose, it is possible to have U_{13} close to the present experimental bound together with a large value for I , provided the ratio $r \equiv (\Delta m_{21}^2 / \Delta m_{32}^2)^{1/2}$ lies within the lower values of the experimentally allowed range. In this case the observation of leptonic CP violation due to a Dirac-type CP violating phase would be within the reach of future neutrino experiments. Finally, it was shown that in this scheme a sufficient amount of baryon asymmetry of the Universe can be generated through leptogenesis.

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References

- [1] G. C. Branco, J. I. Silva-Marcos and M. N. Rebelo, Phys. Lett. B **237** (1990) 446.
- [2] J. Kalinowski and M. Olechowski, Phys. Lett. B **251** (1990) 584.
- [3] P. M. Fishbane and P. Kaus, Phys. Rev. D **49** (1994) 3612.
- [4] G. C. Branco and J. I. Silva-Marcos, Phys. Lett. B **359** (1995) 166 [arXiv:hep-ph/9507299].
- [5] G. C. Branco, D. Emmanuel-Costa and J. I. Silva-Marcos, Phys. Rev. D **56** (1997) 107 [arXiv:hep-ph/9608477].
- [6] P. Q. Hung and M. Seco, Nucl. Phys. B **653** (2003) 123 [arXiv:hep-ph/0111013].
P. Q. Hung, M. Seco and A. Soddu, arXiv:hep-ph/0311198.
- [7] G. C. Branco, M. N. Rebelo and J. I. Silva-Marcos, Phys. Rev. D **62** (2000) 073004 [arXiv:hep-ph/9906368].
- [8] M. Frigerio and A. Y. Smirnov, Nucl. Phys. B **640** (2002) 233 [arXiv:hep-ph/0202247]; Phys. Rev. D **67** (2003) 013007 [arXiv:hep-ph/0207366]; S. Pascoli and S. T. Petcov, Phys. Lett. B **544** (2002) 239 [arXiv:hep-ph/0205022].
- [9] S. Fukuda *et al.* [Super-Kamiokande Collaboration], Phys. Lett. B **539** (2002) 179 [arXiv:hep-ex/0205075].
- [10] Q. R. Ahmad *et al.* [SNO Collaboration], Phys. Rev. Lett. **89** (2002) 011302 [arXiv:nucl-ex/0204009].
- [11] T. Nakaya [SUPER-KAMIOKANDE Collaboration], eConf **C020620** (2002) SAAT01 [arXiv:hep-ex/0209036].
- [12] M. Apollonio *et al.* [CHOOZ Collaboration], Phys. Lett. B **466** (1999) 415 [arXiv:hep-ex/9907037].
- [13] K. Eguchi *et al.* [KamLAND Collaboration], Phys. Rev. Lett. **90** (2003) 021802 [arXiv:hep-ex/0212021].
- [14] H. V. Klapdor-Kleingrothaus, A. Dietz, H. L. Harney and I. V. Krivosheina, Mod. Phys. Lett. A **16** (2001) 2409 [arXiv:hep-ph/0201231]; H. V. Klapdor-Kleingrothaus [Heidelberg–Moscow Collaboration], arXiv:hep-ph/0307330, to appear in the proceedings of 4th Workshop on Neutrino Oscillations and their Origin (NOON2003), Kanazawa, Japan, 10–14 Feb. 2003.

- [15] C. L. Bennett *et al.*, *Astrophys. J. Suppl.* **148** (2003) 1 [arXiv:astro-ph/0302207].
- [16] P. H. Frampton, S. L. Glashow and D. Marfatia, *Phys. Lett. B* **536** (2002) 79 [arXiv:hep-ph/0201008].
- [17] H. Nishiura, K. Matsuda and T. Fukuyama, *Phys. Rev. D* **60** (1999) 013006 [arXiv:hep-ph/9902385]; K. Matsuda, T. Fukuyama and H. Nishiura, *Phys. Rev. D* **61** (2000) 053001 [arXiv:hep-ph/9906433]; P. H. Frampton, S. L. Glashow and T. Yanagida, *Phys. Lett. B* **548** (2002) 119 [arXiv:hep-ph/0208157]; S. Kaneko and M. Tanimoto, *Phys. Lett. B* **551** (2003) 127 [arXiv:hep-ph/0210155]; Z. z. Xing, *Phys. Lett. B* **550** (2002) 178 [arXiv:hep-ph/0210276]; G. C. Branco, R. Gonzalez Felipe, F. R. Joaquim, I. Masina, M. N. Rebelo and C. A. Savoy, *Phys. Rev. D* **67** (2003) 073025 [arXiv:hep-ph/0211001]; M. C. Chen and K. T. Mahanthappa, *Phys. Rev. D* **68** (2003) 017301 [arXiv:hep-ph/0212375]; Z. z. Xing, arXiv:hep-ph/0307007.
- [18] G. C. Branco, R. Gonzalez Felipe, F. R. Joaquim and T. Yanagida, *Phys. Lett. B* **562** (2003) 265 [arXiv:hep-ph/0212341]; T. Endoh, T. Morozumi and Z. h. Xiong, arXiv:hep-ph/0308276.
- [19] M. Fukugita and T. Yanagida, *Phys. Lett. B* **174** (1986) 45.
- [20] B. Pontecorvo, *Sov. Phys. JETP* **6** (1957) 429 [*Zh. Eksp. Teor. Fiz.* **33** (1957) 549]; Z. Maki, M. Nakagawa and S. Sakata, *Prog. Theor. Phys.* **28** (1962) 870.
- [21] K. Hagiwara *et al.* [Particle Data Group], *Phys. Rev. D* **66** (2002) 010001.
- [22] T. Endoh, T. Morozumi, T. Onogi and A. Purwanto, *Phys. Rev. D* **64** (2001) 013006 [Erratum, *ibid.* **64** (2001) 059904] [arXiv:hep-ph/0012345].
- [23] G. C. Branco, T. Morozumi, B. M. Nobre and M. N. Rebelo, *Nucl. Phys. B* **617** (2001) 475 [arXiv:hep-ph/0107164].
- [24] M. N. Rebelo, *Phys. Rev. D* **67** (2003) 013008 [arXiv:hep-ph/0207236].
- [25] E. K. Akhmedov, G. C. Branco, F. R. Joaquim and J. I. Silva-Marcos, *Phys. Lett. B* **498** (2001) 237 [arXiv:hep-ph/0008010].
- [26] M. C. Gonzalez-Garcia and C. Pena-Garay, arXiv:hep-ph/0306001.
- [27] W. Buchmüller, P. Di Bari and M. Plümacher, *Nucl. Phys. B* **643** (2002) 367 [arXiv:hep-ph/0205349]; *Phys. Lett. B* **547** (2002) 128 [arXiv:hep-ph/0209301]; *Nucl. Phys. B* **665** (2003) 445 [arXiv:hep-ph/0302092].
- [28] L. Covi, E. Roulet and F. Vissani, *Phys. Lett. B* **384** (1996) 169 [arXiv:hep-ph/9605319]; M. Flanz, E. A. Paschos and U. Sarkar, *Phys. Lett. B* **345** (1995) 248 [Erratum, *ibid.* **382** (1996) 447] [arXiv:hep-ph/9411366]; M. Plümacher, *Z. Phys. C* **74** (1997) 549 [arXiv:hep-ph/9604229].
- [29] E. W. Kolb and M. S. Turner, “The Early Universe,” Addison-Wesley (1990) 547 p. (Frontiers in Physics, 69).

- [30] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **155** (1985) 36.
- [31] S. Y. Khlebnikov and M. E. Shaposhnikov, Nucl. Phys. B **308** (1988) 885.
- [32] J. A. Harvey and M. S. Turner, Phys. Rev. D **42** (1990) 3344.
- [33] W. Fischler, G. F. Giudice, R. G. Leigh and S. Paban, Phys. Lett. B **258** (1991) 45.
- [34] W. Buchmüller and T. Yanagida, Phys. Lett. B **302** (1993) 240.
- [35] W. Buchmüller, P. Di Bari and M. Plümacher (2003) in Ref. [27]. For other recent detailed studies on thermal leptogenesis see: S. Davidson, arXiv:hep-ph/0304120; A. Pilaftsis and T. E. J. Underwood, arXiv:hep-ph/0309342; G. F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, arXiv:hep-ph/0310123.

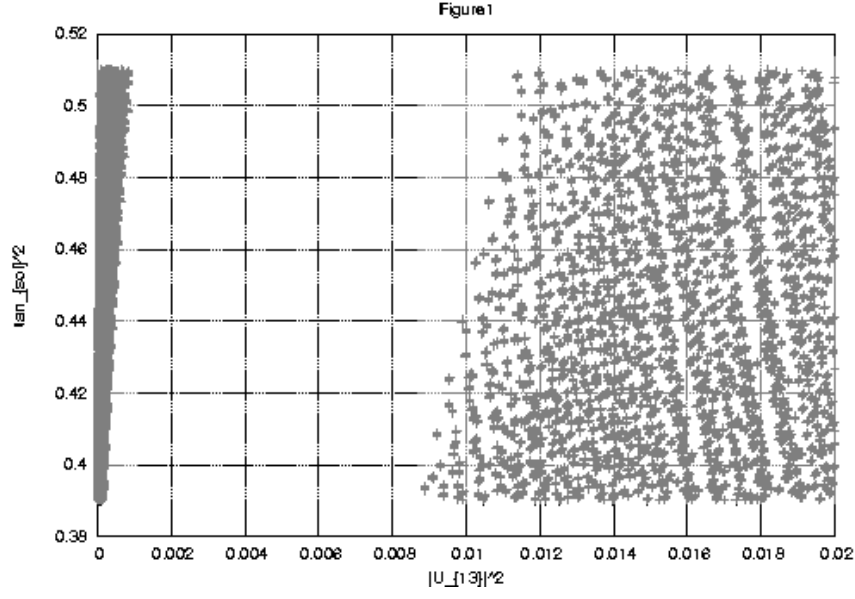


Fig. 1. Region allowed for $|U_{13}|^2$ and $\tan^2(\theta_{sol})$.

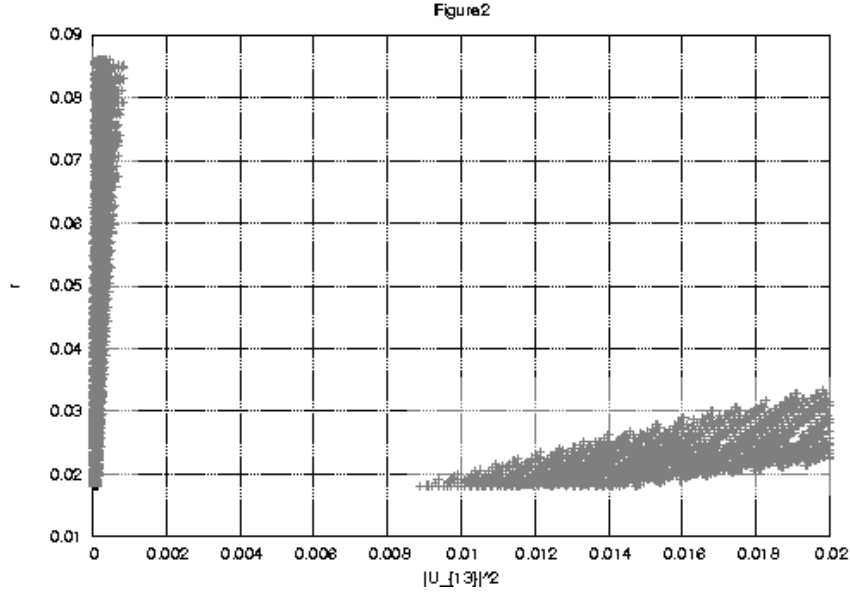


Fig. 2. Region allowed for $|U_{13}|^2$ and the mass ratio $r = (\Delta m_{21}^2 / \Delta m_{23}^2)^{(1/2)}$.

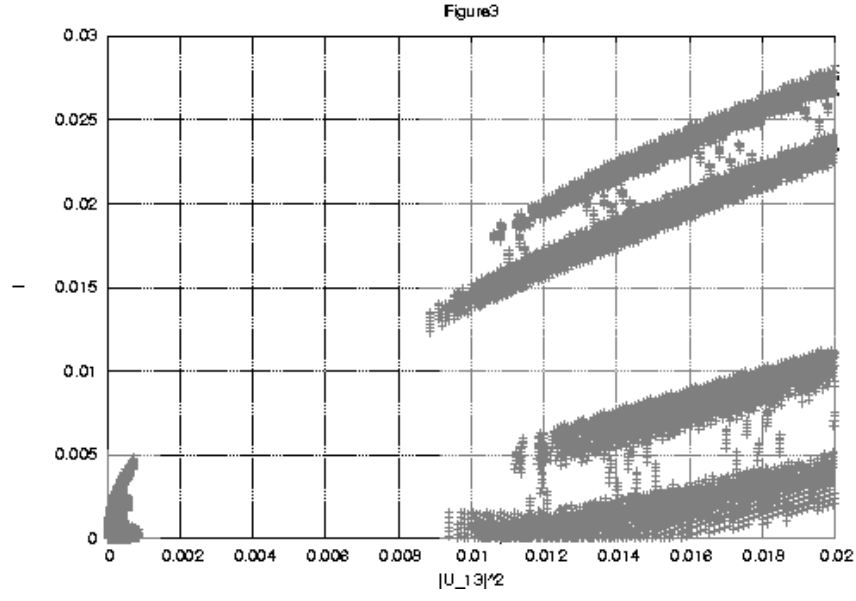


Fig. 3. Region allowed for $|U_{13}|^2$ and the invariant I .

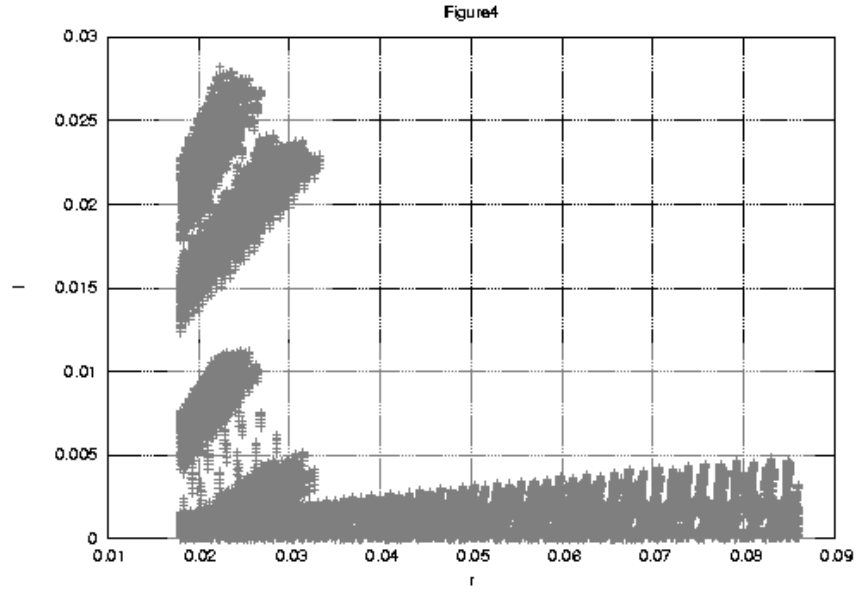


Fig. 4. Region allowed for the mass ratio $r = (\Delta m_{21}^2 / \Delta m_{23}^2)^{(1/2)}$ and I .

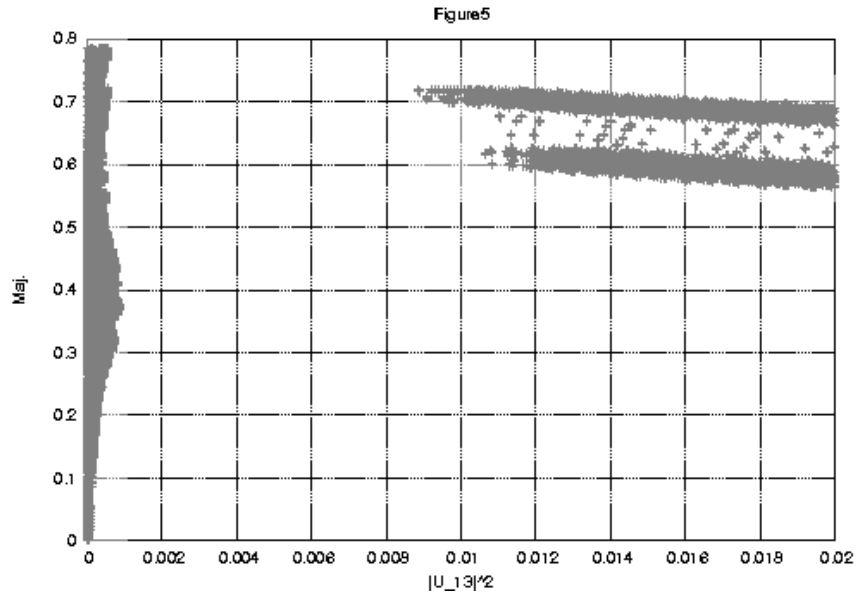


Fig. 5. Region allowed for $|U_{13}|^2$ and the Majorana phase combination $\delta - \alpha$.